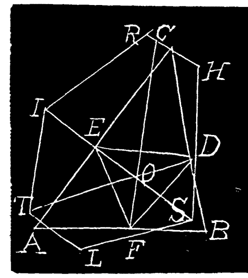
Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let ABC be the triangle, DEF the stretched string, P the force exerted by the string, O the in-center of DEF. Then the resultant of the forces in DE, DF is $2P\cos\frac{1}{2}A$, acting from D through O. Let OT represent the this force. The resultant of the forces in FE, FD is $2P\cos\frac{1}{2}C$ acting from F through O. Let OR represent this force. The resultant of the forces in EF, ED is $2P\cos\frac{1}{2}B$ acting from E through O. Let OS represent this force. OT can be replaced by two parallel forces equal to $\frac{1}{2}OT$ acting at B and C, respectively; OR, by two parallel forces equal to $\frac{1}{2}OR$ acting at A and B, respectively; and OS, by two parallel forces equal to $\frac{1}{2}OS$ acting at A and C, respectively. Completing the parallelograms, we get OH the resultant of OR and OS, and $\frac{1}{2}OH$ represents the reaction at A. OI is the resultant of OR and OT while $\frac{1}{2}OI$ represents the reaction at B. OL is the resultant of OT and OS, and $\frac{1}{2}OL$ represents the reaction at C.



AVERAGE AND PROBABILITY.

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194. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

What is the mean value of the triangle formed by joining three points taken at random on the circumference of a circle?

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let OC=diameter of given circle$=2a$. Let the point P be fixed, draw PO perpendicular to OC, and draw OA and OB. Let $\angle POA=\theta>\frac{1}{2}\pi$, $\angle POB=\phi<\frac{1}{2}\pi$.

$\therefore OA=2a\sin\theta$, and $OB=2a\sin\phi$. Area $OAB=2a^2\sin\theta\sin\phi\sin(\theta-\phi)$. Average area$=\triangle$,

$$=\frac{\int_0^\pi\int_0^\theta 2a^2\sin\theta\sin\phi\sin(\theta-\phi)d\theta\, d\phi}{\int_0^\pi\int_0^\theta d\theta\, d\phi}$$

$$=\frac{4a^2}{\pi^2}\int_0^\pi\int_0^\theta \sin\theta\sin\phi\sin(\theta-\phi)d\theta\, d\phi=\frac{2a^2}{\pi^2}\int_0^\pi(\sin^2\theta-\theta\sin\theta\cos\theta)d\theta$$

$$=\frac{3a^2}{2\pi}.$$